Wire Heat Transfer for CSO Dewars

F. Rice

5/26/2010

Part I: Heat transfer for wires carrying negligible current

Definitions for the Thermal Conductivity and Resistivity of Manganin and Phosphor Bronze

Data Source: **http://www.lakeshore.com/temp/acc/am_wirets.html**

In[3]:=
```
Needs["Units`"]
```

Here is the thermal conductivity data from the Lakeshore web page.

In[4]:=
```
ManganinTcTable = {
    (* {T (Kelvin), ρ (Watt/(Meter Kelvin))} *)
    {.1, .006},
    {.4, .02},
    {1., .06},
    {4., .5},
    {10., 2.0},
    {20., 3.3},
    {80., 13.},
    {150., 16.},
    {300., 22.}
   };
```

In[5]:=
```
PBronzeTcTable = {
    (* {T (Kelvin), ρ (Watt/(Meter Kelvin))} *)
    {1., .22},
    {4., 1.6},
    {10., 4.6},
    {20., 10.0},
    {80., 25.},
    {150., 34.},
    {300., 48.}
   };
```

- **Functions for thermal conductivity and a thermal integral**

Interpolate the thermal conductivity data to give a function. Generate a thermal integral function from a given temperature to the shield temperature of 15K.

In[6]:=
```
SetMaterial["Manganin"] :=
 (
  Clear[ThermalConductivityTable, TC];
  ThermalConductivityTable = ManganinTcTable;
  MakeTC;
  MakeTI;
  rho = rhoM;
  material = "Manganin";
 )
```

In[7]:=
```
SetMaterial["Phosphor Bronze"] :=
 (
  Clear[ThermalConductivityTable, TC];
  ThermalConductivityTable = PBronzeTcTable;
  MakeTC;
  MakeTI;
  rho = rhoPB;
  material = "Phosphor Bronze";
 )
```

In[8]:=
```
MakeTC := (
  Clear[TC];
  TC = Interpolation[ThermalConductivityTable, InterpolationOrder → 1];
  (* Watt/(Meter Kelvin) *)
 )
```

In[257]:=
```
(* this is a straightforward thermal integral implementation,
 but not very fast *)
 Clear[TI];
 TI[t_ ?NumberQ(* Kelvin *)] := NIntegrate[TC[tt], {tt, t, 15.0},
   Method → {"Trapezoidal", "SymbolicProcessing" → 0}](* Watt/Meter *)
 TI[tL_?NumberQ (* Kelvin *), tH_ ?NumberQ(* Kelvin *)] :=
  NIntegrate[TC[tt], {tt, 1. tL, 1. tH},
   Method → {"Trapezoidal", "SymbolicProcessing" → 0}](* Watt/Meter *)
```

In[9]:=
```
MakeTI :=
 (
  (* this is an analytic thermal integral implementation to a linear
   interpolation of κ(T), over 15x faster than the above numerical integration *)
  Clear[TItable, TI];
  TItable =
   Block[{list1, list2, list3, mb, mbInt},
    mb[{x1_, y1_}, {x2_, y2_}] := {y1 - y2, -x2 y1 + x1 y2}/(x1 - x2);
    mbInt[{{x1_, y1_}, {x2_, y2_}}] := Block[{m, b, xx},
      {m, b} = mb[{x1, y1}, {x2, y2}];
      {Evaluate[FullSimplify[Together[Integrate[m xx + b, {xx, x1, x2}]]]],
       Evaluate[Function[{x},
         Evaluate[FullSimplify[Together[Integrate[m xx + b, {xx, x, x2}]]]]], Evaluate[
        Function[{x}, Evaluate[FullSimplify[Together[Integrate[m xx + b, {xx, x1, x}]]]]]}
      ];
    list1 = {Most[ThermalConductivityTable[[All, 1]] ],
       Rest[ThermalConductivityTable[[All, 1]]]} // Transpose;
    list2 = {Most[ThermalConductivityTable], Rest[ThermalConductivityTable]} // Transpose;
    Transpose[{list1, mbInt /@ list2}]
   ];

  TI[xlow_?NumberQ, xhigh_?NumberQ] := Block[{i1, i2, sum},
    {i1, i2} = Join[Last[Position[TItable, {{x_, _}, {__}} /; x <= xlow]],
      First[Position[TItable, {{_, x_}, {__}} /; x >= xhigh]]];
    If[i1 == i2, Return[TItable[[i1, 2, 3]][xhigh] - TItable[[i1, 2, 3]][xlow]]];
    sum = TItable[[i1, 2, 2]][xlow] + TItable[[i2, 2, 3]][xhigh];
    For[i1++, i1 < i2, i1++, sum += TItable[[i1, 2, 1]]];
    sum
   ];
  TI[xlow_?NumberQ] := TI[xlow, 15.];
  (* The thermal integral value for 4K-15K: *)
  TIntegral = TI[4.0] (* Watt/Meter *);
 )
```

■ Resistivity of Manganin at 4K is about 90% of its room temperature value

The resistivity of Manganin is a *very* slowly varying function of temperature, so it is essentially constant between 4k and 15k.

Data Source: **http://www.iceoxford.com/Cryogenic-spares/Wiring.htm**

In[10]:=
```
ratio = Mean[{24 / 27.,
   37 / 41.,
   83 / 93.,
   147 / 164.}]
```

Out[10]= 0.895036

Data Source: **http://www.lakeshore.com/temp/acc/am_wirets.html**

The resistivity of Manganin at 4K (in $\mu\Omega$ cm):

In[11]:=
```
rhoM = ratio 48 (* Micro Ohm Centi Meter *)
```

Out[11]=
```
42.9617
```

■ Resistivity of Phosphor Bronze is very nearly constant between 4K and 50K

Data Source: **http://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/20090032058_ 2009032566.pdf**

The resistivity of Phosphor Bronze at 4K (in $\mu\Omega$ cm):

In[12]:=
```
rhoPB = 8.8 (* Micro Ohm Centi Meter *)
```

Out[12]=
```
8.8
```

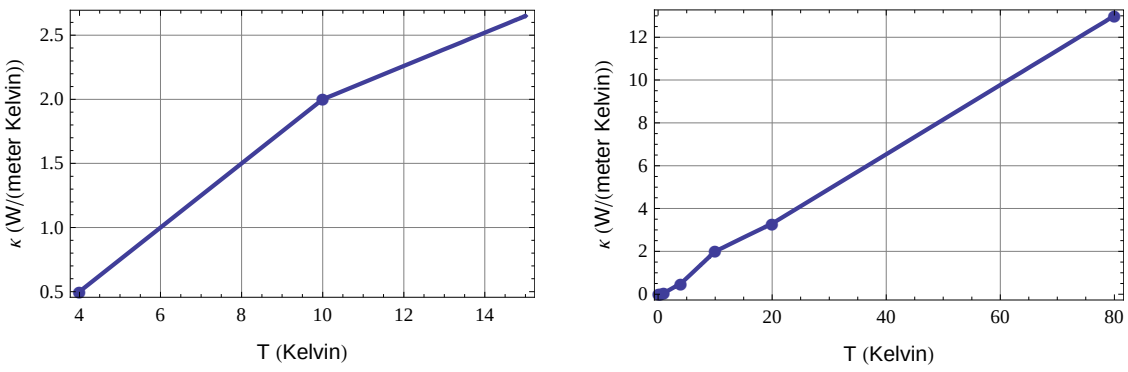
In[13]:=
```
SetMaterial["Manganin"]
```

■ Thermal conductivity Plot

In[14]:=
```
SetMaterial["Manganin"]
```

In[15]:=
```
Labeled[Row[{
   Show[Plot[TC[t], {t, 4, 15}, Frame → True, GridLines → Automatic, PlotStyle → Thick,
     FrameLabel → {Style["T (Kelvin)", Larger, FontFamily → "Helvetica"],
       Style["κ (W/(meter Kelvin))", Larger, FontFamily → "Helvetica"]},
     ImageSize → 250], ListPlot[ThermalConductivityTable, PlotMarkers → Automatic]],
   Show[Plot[TC[t], {t, 1, 80}, Frame → True, GridLines → Automatic, PlotStyle → Thick,
     FrameLabel → {Style["T (Kelvin)", Larger, FontFamily → "Helvetica"],
       Style["κ (W/(meter Kelvin))", Larger, FontFamily → "Helvetica"]},
     ImageSize → 250], ListPlot[ThermalConductivityTable, PlotMarkers → Automatic]]
 }, "\t"],
Style["Thermal Conductivity for Manganin\n", Larger, Italic, FontFamily → "Times"], Top]
```

Out[15]=



Heat transfer for a wire with negligible current - 4K to 15K

■ Function for the area of a wire with diameter in mils

This function calculates the area of a wire (in square meters) from its diameter in mils

```
In[16]:= area[dia_ (* mils *)] :=
          dia^2 Evaluate[(π / 4) Convert[Mil^2 / Meter^2, 1.0]] (* square meters *)

In[17]:= area[d]

Out[17]= 5.06707 × 10^-10 d^2
```

■ A function to calculate the L/A ratio in 1/Meter for a specified wire bundle

```
In[18]:= LbyA[n_ (* number of wires *), Len_ (* inches *), dia_ (* mils *)] :=
          Len / (n area[dia]) Evaluate[Convert[ Inch / Meter, 1]] / Meter

         LbyA[Len_ (* inches *), dia_ (* mils *)] := LbyA[1, Len, dia]

In[20]:= LbyA[8(* number of wires *), 4. (* inches *), 5.(* mils *)]

Out[20]= 1.00255 × 10^6 / Meter
```

■ Function for heat transfer v. wire diameter (mils) and length (inches) from 4K to 15K, output in microwatts

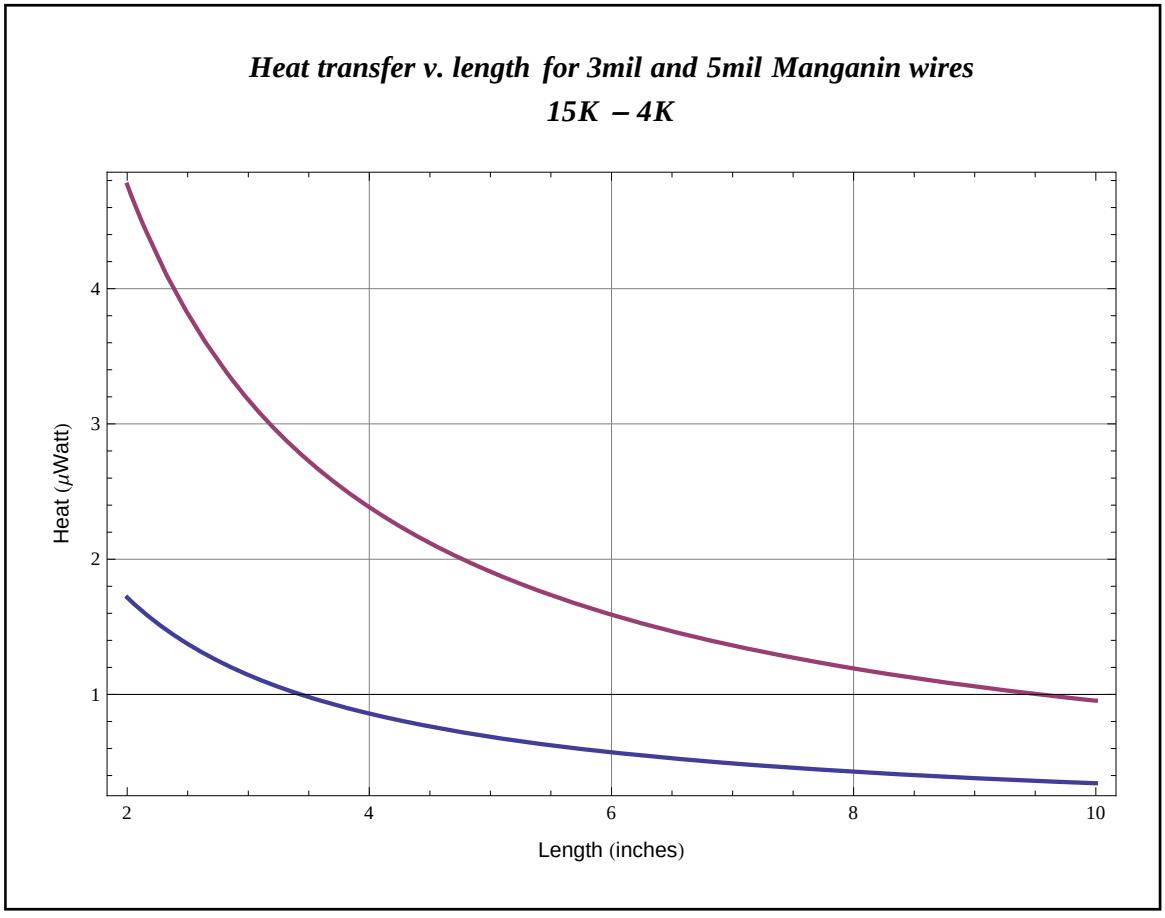
```
In[21]:= Heat[ratio_(* L/A in 1/Meter *)] := 10^6 (Micro Watt) TIntegral / (ratio Meter)(* μwatts *)

         Heat["T", ratio_(* L/A in 1/Meter *), Tlow_(* K *), Thigh_(* K *)] :=
          10^6 (Micro Watt) TI[Tlow, Thigh] / (ratio Meter)(* μwatts *)


         Heat[n_ (* number of wires *), len_ (* inches *), dia_ (* mils *)] :=
          10^6 (Micro Watt) n area[dia] TIntegral / (len Evaluate[ Convert[Inch / Meter, 1.0]])

         Heat[len_ (* inches *), dia_ (* mils *)] := Heat[1, len, dia]
```

■ Compare 3mil and 5mil Manganin wires

```
In[25]:= SetMaterial["Manganin"]
```

In[26]:=
```
Framed[
 Plot[{Heat[x, 3.0] / (Micro Watt), Heat[x, 5.0] / (Micro Watt)},
  {x, 2, 10}, Frame → True, GridLines → Automatic, PlotStyle → Thick,
  PlotLabel → Style["Heat transfer v. length for 3mil and 5mil Manganin wires\n15K - 4K\n",
    Larger, Bold, Italic],
  FrameLabel → {Style["Length (inches)", Larger, FontFamily → "Helvetica"],
    Style["Heat (μWatt)", Larger, FontFamily → "Helvetica"]},
  ImageSize → 500], FrameMargins → 20, Background → White
]
```

Out[26]=



In[27]:=
```
Framed[Column[{
   Style["Heat transfer v. length for Manganin wires",
    Larger, Bold, Italic, FontFamily → "Times"],
   Style["15K – 4K\n", Bold, Italic, FontFamily → "Times"],
   Row[{
     Framed[Labeled[
       Style[TableForm[Table[{x, 10.^-2 Round[10.^2 Heat[x, 3.] / (Micro Watt)]}, {x, 1, 12}],
         TableHeadings → {None, {Style["inches", Italic, Larger, FontFamily → "Times"],
            Style["μWatt", Italic, Larger, FontFamily → "Times"]}},
         TableAlignments → Center, TableSpacing → {1.2, 4}], FontFamily → "Helvetica"],
       Style["3 mil\n", Bold, Italic, Larger, FontFamily → "Times", Top],
      FrameMargins → 15],
     Framed[Labeled[
       Style[TableForm[Table[{x, 10.^-2 Round[10.^2 Heat[x, 5.] / (Micro Watt)]}, {x, 1, 12}],
         TableHeadings → {None, {Style["inches", Italic, Larger, FontFamily → "Times"],
            Style["μWatt", Italic, Larger, FontFamily → "Times"]}},
         TableAlignments → Center, TableSpacing → {1.2, 4}], FontFamily → "Helvetica"],
       Style["5 mil\n", Bold, Italic, Larger, FontFamily → "Times", Top],
      FrameMargins → 15]
    }, "\t\t"]
  }, Center],
 FrameMargins → 15, Background → White]
```

Out[27]=

Heat transfer v. length for Manganin wires

15K – 4K

3 mil

inches	*μWatt*
1	3.43
2	1.72
3	1.14
4	0.86
5	0.69
6	0.57
7	0.49
8	0.43
9	0.38
10	0.34
11	0.31
12	0.29

5 mil

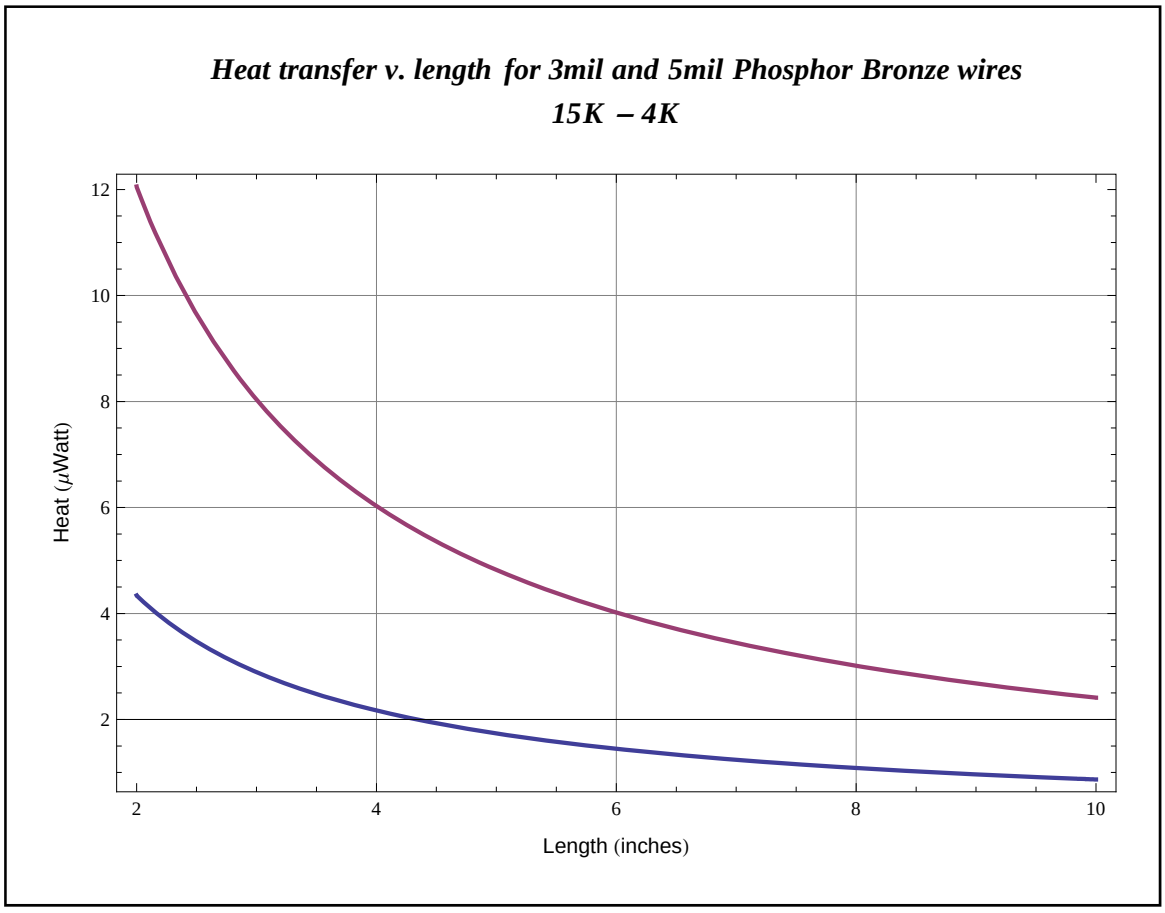
inches	*μWatt*
1	9.54
2	4.77
3	3.18
4	2.38
5	1.91
6	1.59
7	1.36
8	1.19
9	1.06
10	0.95
11	0.87
12	0.79

■ Compare 3mil and 5mil Phosphor Bronze wires

In[28]:=
```
SetMaterial["Phosphor Bronze"]
```

In[29]:=
```
Framed[
 Plot[{Heat[x, 3.0] / (Micro Watt), Heat[x, 5.0] / (Micro Watt)},
  {x, 2, 10}, Frame → True, GridLines → Automatic, PlotStyle → Thick,
  PlotLabel → Style[
    "Heat transfer v. length for 3mil and 5mil Phosphor Bronze wires\n15K - 4K\n",
    Larger, Bold, Italic],
  FrameLabel → {Style["Length (inches)", Larger, FontFamily → "Helvetica"],
    Style["Heat (μWatt)", Larger, FontFamily → "Helvetica"]},
  ImageSize → 500], FrameMargins → 20, Background → White
]
```

Out[29]=



In[30]:=
```
Framed[Column[{
   Style["Heat transfer v. length for Phosphor Bronze wires",
    Larger, Bold, Italic, FontFamily → "Times"],
   Style["15K – 4K\n", Bold, Italic, FontFamily → "Times"],
   Row[{
     Framed[Labeled[
       Style[TableForm[Table[{x, 10.^-2 Round[10.^2 Heat[x, 3.] / (Micro Watt)]}, {x, 1, 12}],
         TableHeadings → {None, {Style["inches", Italic, Larger, FontFamily → "Times"],
            Style["μWatt", Italic, Larger, FontFamily → "Times"]}},
         TableAlignments → Center, TableSpacing → {1.2, 4}], FontFamily → "Helvetica"],
       Style["3 mil\n", Bold, Italic, Larger, FontFamily → "Times", Top],
      FrameMargins → 15],
     Framed[Labeled[
       Style[TableForm[Table[{x, 10.^-2 Round[10.^2 Heat[x, 5.] / (Micro Watt)]}, {x, 1, 12}],
         TableHeadings → {None, {Style["inches", Italic, Larger, FontFamily → "Times"],
            Style["μWatt", Italic, Larger, FontFamily → "Times"]}},
         TableAlignments → Center, TableSpacing → {1.2, 4}], FontFamily → "Helvetica"],
       Style["5 mil\n", Bold, Italic, Larger, FontFamily → "Times", Top],
      FrameMargins → 15]
    }, "\t\t"]
  }, Center],
 FrameMargins → 15, Background → White]
```

Out[30]=

Heat transfer v. length for Phosphor Bronze wires

15K – 4K

3 mil

inches	*μWatt*
1	8.68
2	4.34
3	2.89
4	2.17
5	1.74
6	1.45
7	1.24
8	1.09
9	0.96
10	0.87
11	0.79
12	0.72

5 mil

inches	*μWatt*
1	24.11
2	12.06
3	8.04
4	6.03
5	4.82
6	4.02
7	3.44
8	3.01
9	2.68
10	2.41
11	2.19
12	2.01